Disorder-Induced Phase Control in Superfluid Fermi-Bose Mixtures

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We consider a mixture of a superfluid Fermi gas of ultracold atoms and a Bose-Einstein condensate of molecules possessing a continuous $U(1)$ (relative phase) symmetry. We study the effects that a spatially random photo-associative-dissociative symmetry breaking coupling of the systems. Such coupling allows to control the relative phase between a superfluid order parameter of the Fermi system and the condensate wavefunction of molecules for temperatures below the BCS critical temperature. The presented mechanism of phase control belongs to the general class of disorder-induced order phenomena that rely on breaking of continuous symmetry.

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Of all the studies of the physics of ultracold atoms, fermionic systems and in particular Fermi superfluids have recently received a lot of attention. The most famous example in this context is the BEC-BCS crossover. In this example, fermions are transformed between the superconducting state in which they form Bardeen-Cooper-Schriefer (BCS) pairs and the Bose-Einstein Condensate (BEC) made of diatomic molecules of the same fermions (for reviews see [1, 2, 3]). Another field of particularly high activity has recently been the study of disorder in ultracold gases. Since its proposal [4], several experimental and theoretical groups have focused on this subject [5, 6]. Very recently, these studies culminated in the seminal experimental observation of Anderson localization of matter waves in a BEC with a disordered potential [7, 8] and in signatures of the Bose glass state of an ultracold gas in an optical lattice [9].

We have proposed an additional type of disorder effects that can be realized with ultracold atoms, called *disorder induced order* [10]. Generally speaking, the effect can be observed in systems possessing a continuous symmetry which, in accordance with the Mermin-Wagner-Hohenberg theorem, prevents long range order in low dimensions [11]. Comparing models without and with continuous symmetries, we therefore observe that the latter have lower critical temperatures in any dimension. Adding disorder that breaks the continuous symmetry leads to competing mechanisms in such systems: naturally, disorder acts against ordering whereas symmetry breaking increases the tendency to order. We have studied this effect for a classical XY model in a random X oriented magnetic field and have shown rigorously the appearance of spontaneous magnetization in the Y direction at $T = 0$. Also, we have presented strong evidence for the survival of magnetization at $T > 0$ in the limit of small disorder. These ideas were then applied to quantum systems: a two component Bose gas in an optical lattice [10] and a two component BEC with random Raman coupling [12]. It is worth noticing that similar effects can be found in other areas of physics (see for instance [13], for classical mean field modelssee [14]). Also, the effect is robust enough to appear not only in random, but also in oscillating symmetry breaking perturbations.

The main goal of the present letter is to introduce these new ideas to the field of superfluid Fermi gases that posses a phase $U(1)$ symmetry. We show that a Fermi superfluid coupled to a molecular BEC via a random, symmetry breaking photo-associating-dissociating coupling undergoes relative phase ordering, so that the phase of the order parameter can be efficiently controlled by the phases of the coupling. Within a BCS-like theory [15], we show rigorously that for small disorder the effect occurs practically for all temperatures below the superfluid transition temperature.

We consider a mixture of fermions in two different internal states interacting via attractive zero-range potential in a volume V in 3D with a Hamiltonian

$$H_F = \int d\mathbf{r} \left[\hat{\psi}_j^\dagger \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu \right) \hat{\psi}_j - g \hat{\psi}_1^\dagger \hat{\psi}_2^\dagger \hat{\psi}_2 \hat{\psi}_1 \right], \quad (1)$$

where the chemical potential μ fixes the average density n, $g = 4\pi\hbar^2|a|/m$ (with s-wave scattering length a) determines strength of the interactions and the $\hat{\psi}_j$ stand for fermionic field operators. Moreover, we assume the presence of a Bose-Einstein condensate (BEC) of molecular dimers consisting of the two fermionic species and a (weak) coupling that transforms them into fermion pairs and vice versa. Experimentally, we can sweep a mixture of fermions with two different internal states over a Feshbach resonance, which leaves us with a BEC of diatomic molecules and unbound fermions. Then, we approach a second Feshbach resonance which turns the previously unbound fermions into BCS pairs without affecting the molecular BEC. The coupling between the

molecules and the fermions can be realized through photoassociation and photodissociation, respectively. Taking the limit of a large BEC, we do not need to consider the dynamics of the BEC because the effect of the weak coupling with the fermions on BEC is negligible. Following these considerations, the Hamiltonian (1) has to be supplemented by one term only: the coupling between the fermions and the BEC which we approximate as $\int d\mathbf{r}\left[\gamma^*(\mathbf{r})\hat{\psi}_d^\dagger\hat{\psi}_1\hat{\psi}_2+\gamma(\mathbf{r})\hat{\psi}_2^\dagger\hat{\psi}_1^\dagger\hat{\psi}_d\right]\approx \sqrt{n_d}\int d\mathbf{r}\left[\gamma^*(\mathbf{r})\hat{\psi}_1\hat{\psi}_2+\gamma(\mathbf{r})\hat{\psi}_2^\dagger\hat{\psi}_1^\dagger\right]$, where the bosonic filed operator $\hat{\psi}_d$ for molecules is substituted by a real-valued condensate wavefunction which for a homogeneous case considered here is squared root of the density of dimers $\sqrt{n_d}$. The full Hamiltonian therefore reads

$$H=H_F+\int d\mathbf{r}\left[\Gamma^*(\mathbf{r})\hat{\psi}_1\hat{\psi}_2+\Gamma(\mathbf{r})\hat{\psi}_2^\dagger\hat{\psi}_1^\dagger\right], \quad (2)$$

with $\Gamma(\mathbf{r})=\tilde{\Gamma}(\mathbf{r})e^{-i\varphi_\Gamma}=\sqrt{n_d}\gamma(\mathbf{r})$. We assume that the transfer process is realized such that $\tilde{\Gamma}(\mathbf{r})$ is real, changes randomly in space and is constant in time, $\int d\mathbf{r}\tilde{\Gamma}(\mathbf{r})=0$ and φ_Γ is a real number. We will show that for $\varphi_\Gamma=0$ the relative phase between the condensate wavefunction of molecules and the paring function of the superfluid fermions is fixed to $\pi/2$ (or $-\pi/2$). Then we show that we can control the relative phase and fix it to any value by changing a control parameter φ_Γ.

For $\Gamma=0$ and in the weak coupling limit, i.e. $g\to 0$, we deal with a Fermi system which for T less than the critical temperature $T_c=8e^{\eta}e^{-2}\pi^{-1}T_F e^{-\pi/2k_F|a|}$ (where $k_BT_F=\varepsilon_F=\hbar^2k_F^2/2m=\hbar^2(3\pi^2n)^{2/3}/2m$ and $\eta=0.5772$) reveals a transition to a superfluid phase (BCS state) which is indicated by a non-vanishing pairing function (order parameter) $\Delta=g\langle\hat{\psi}_2\hat{\psi}_1\rangle$ [15, 16]. In the general case (i.e. including non-zero Γ) the pairing function is given in terms of solutions of Bogoliubov-de Gennes (BdG) equations, i.e. $\Delta=g\sum_n u_n v_n^*[1-2f(E_n)]$, where

$$\begin{bmatrix} -\frac{\hbar^2\nabla^2}{2m}-\mu+W & \Delta+\Gamma \\ \Delta^*+\Gamma^* & \frac{\hbar^2\nabla^2}{2m}+\mu-W\end{bmatrix}\begin{bmatrix}u_n\\ v_n\end{bmatrix}=E_n\begin{bmatrix}u_n\\ v_n\end{bmatrix}, \quad (3)$$

with a Hartree-Fock term $W=-g\langle\hat{\psi}_1^\dagger\hat{\psi}_1\rangle=-g\langle\hat{\psi}_2^\dagger\hat{\psi}_2\rangle=-g\sum_n\left(|u_n|^2f(E_n)+|v_n|^2[1-f(E_n)]\right)$ and $f(E_n)=\left(e^{E_n/k_BT}+1\right)^{-1}$ [15].

If the transfer process is absent (i.e. $\Gamma=0$) the system (2) is invariant under global gauge transformation, i.e. $\hat{\psi}_j\to e^{i\varphi/2}\hat{\psi}_j$, which implies that if $\{u_n,v_n\}$ are solutions of the BdG equations for Δ, then $\{e^{i\varphi/2}u_n,e^{-i\varphi/2}v_n\}$ are the solutions corresponding to $e^{i\varphi}\Delta$. This continuous symmetry is broken when the transfer process is turned on, as can be seen through (2). Then the phase of the pairing function becomes relevant because it is the relative phase with respect to the (real-valued) condensate wavefunction of dimers.

We begin with an analysis of the $\varphi_\Gamma=0$ case, i.e. $\Gamma=\tilde{\Gamma}$ is real. Let us assume that for $\Gamma=0$ and for some temperature T we have a non-zero pairing function which is chosen to be real and positive, $\Delta_0>0$. When we turn on Γ but $|\Gamma(\mathbf{r})|\ll\Delta_0$ one may expect that it results in a new pairing function where $\Delta(\mathbf{r})\approx\Delta_0e^{i\varphi(\mathbf{r})}$. That is, any non-zero Γ has a dramatic effect on the phase because without the transfer process the system is degenerated with respect to the choice of φ. On the other hand an infinitesimally small Γ is not able to change $|\Delta(\mathbf{r})|$ because this would cost energy. Moreover, we may expect that $\varphi(\mathbf{r})$ oscillates around some average value φ_0 with small amplitude because we assume that $\Gamma(\mathbf{r})$ fluctuates around zero with infinitesimally small variance. Under these assumptions we can observe that $|\varphi_0|=\pi/2$. Indeed, let us neglect the Hartree-Fock term W (which is not essential for Fermi superfluidity) and calculate the difference of the thermodynamic potentials for superfluid and normal phase

$$\begin{aligned}\Omega_s-\Omega_0 &= \int_0^1\frac{d\lambda}{\lambda}\langle\lambda H_1\rangle_\lambda \\ &\approx -\int_0^g\frac{dg'}{g'^2}\int d\mathbf{r}\left[|\Delta|^2+\frac{2g'}{g}\tilde{\Gamma}|\Delta|\cos\varphi\right] \end{aligned} \quad (4)$$

where $H_1=\int d\mathbf{r}\left[\tilde{\Gamma}(\hat{\psi}_1\hat{\psi}_2+\hat{\psi}_2^\dagger\hat{\psi}_1^\dagger)-g\hat{\psi}_1^\dagger\hat{\psi}_2^\dagger\hat{\psi}_2\hat{\psi}_1\right]$ [15]. According to the assumptions, for g' close to g, $|\Delta(\mathbf{r})|$ is constant and $\cos\varphi(\mathbf{r})\approx\cos\varphi_0-\sin\varphi_0\delta\varphi(\mathbf{r})$. Then

$$\begin{aligned}&-\int d\mathbf{r}\left[|\Delta|^2+\frac{2g'}{g}\tilde{\Gamma}|\Delta|\cos\varphi\right]\approx \\ &-|\Delta|^2V+\sin\varphi_0\frac{2g'|\Delta|}{g}\int d\mathbf{r}\tilde{\Gamma}\delta\varphi, \end{aligned}\quad (5)$$

and for $\int d\mathbf{r}\tilde{\Gamma}\delta\varphi<0$ the thermodynamic potential is minimized when $\varphi_0=\pi/2$. With the transformation $\delta\varphi\to-\delta\varphi$ and $\varphi_0\to-\pi/2$ we obtain another solution what reflects a symmetry of the system. That is, for a real Γ, if $\Delta(\mathbf{r})$ corresponds to solutions of (3) then solutions of complex conjugate BdG equations results in a new pairing function equal $\Delta^*(\mathbf{r})$. In experiments the sign of φ_0 will depend on the preparation and is determined by spontaneous breaking of the $\varphi\to-\varphi$ symmetry.

Having determined φ_0 we would like to estimate fluctuations of the phase of the pairing function $\delta\varphi(\mathbf{r})$. To this end let us employ Ginzburg-Landau (GL) approach [15]. Adapting the Gorkov's derivation of the GL equation [15, 17, 18] (with the standard regularization of the bare interaction g for the case of cold atomic gases) to our problem one obtains

$$\begin{aligned}\nabla^2\Delta=&-\nabla^2\tilde{\Gamma}-\frac{48\pi^2}{7\zeta(3)l_c^2}\left(\frac{2\pi^2\hbar^2}{mk_Fg}+\frac{T_c-T}{T_c}\right)\tilde{\Gamma}\\ &-\frac{48\pi^2}{7\zeta(3)l_c^2}\frac{T_c-T}{T_c}\Delta+\frac{6m^2}{\hbar^4k_F^2}|\Delta+\tilde{\Gamma}|^2(\Delta+\tilde{\Gamma}),\end{aligned}\quad (6)$$

where $l_c = \hbar^2 k_F / m k_B T_c$. Equation (6) is valid for $T_c - T \ll T_c$ and for $\tilde{\Gamma}(\mathbf{r})$ that changes on a scale much larger than l_c (e.g. for $k_F|a| = 0.5$ and $n \sim 10^{14}$ cm^{-3} we get $l_c \sim 4$ μm). For $|\tilde{\Gamma}(\mathbf{r})|$ much smaller than $|\Delta_0(T)|$, i.e. the pairing function in the absence of the transfer process, we may introduce further approximations that allow us to reduce Eq. (6) to

$$|\Delta_0|\nabla^2\delta\varphi(\mathbf{r}) = \nabla^2\tilde{\Gamma}(\mathbf{r}) + \frac{48\pi^2}{7\zeta(3)l_c^2}\left(\frac{2\pi^2\hbar^2}{mk_F g} + \frac{T_c - T}{T_c}\right)\tilde{\Gamma}(\mathbf{r}), \tag{7}$$

where $|\Delta| \approx |\Delta_0|$ and we have chosen $\varphi_0 = \pi/2$ in the expansion $\varphi(\mathbf{r}) \approx \varphi_0 + \delta\varphi(\mathbf{r})$. The solution of (7) reads

$$\delta\varphi(\mathbf{k}) = \frac{\tilde{\Gamma}(\mathbf{k})}{|\Delta_0|} - \frac{48\pi^2}{7\zeta(3)l_c^2|\Delta_0|}\left(\frac{2\pi^2\hbar^2}{mk_F g} + \frac{T_c - T}{T_c}\right)\frac{\tilde{\Gamma}(\mathbf{k})}{|\mathbf{k}|^2}, \tag{8}$$

in the Fourier space.

Now we switch to a general case of complex $\Gamma = \tilde{\Gamma}e^{-i\varphi_\Gamma}$. It is easy to check that if $|\Delta|e^{i\varphi}$ corresponds to solutions of the BdG equations with $\varphi_\Gamma = 0$ then $|\Delta|e^{i(\varphi-\varphi_\Gamma)}$ is related to the solutions for $\varphi_\Gamma \neq 0$. This implies that, if for $\varphi_\Gamma = 0$ we are able to fix the relative phase between the condensate wavefunction of molecules and the pairing function of the superfluid fermions to $\pi/2$ (or $-\pi/2$), then changing φ_Γ allows us to fix it to $\phi_0 = \pi/2 - \varphi_\Gamma$ (or $\phi_0 = -\pi/2 - \varphi_\Gamma$) and phase control emerges.

Assuming that the transfer process with small $|\Gamma(\mathbf{r})|$ results in phase fluctuations of $\Delta(\mathbf{r})$ only, we have shown that the fluctuations occur around $\pi/2 - \varphi_\Gamma$ (or $-\pi/2 - \varphi_\Gamma$) and they are given by Eq. (8). Now we would like to switch to numerical solutions of the BdG equations (where, in contrast to the analytical analysis, we do not neglect the Hartree-Fock term W) to demonstrate that indeed for $|\Gamma| \ll |\Delta_0|$ the fluctuations are small and the predicted phase control is possible. In 3D calculations we regularize the coupling constant g in $\Delta = g\langle\hat{\psi}_2\hat{\psi}_1\rangle$, i.e. $g \to g_{eff}$, according to

$$\frac{1}{g_{eff}} = \frac{1}{g} - \frac{mk_F}{2\pi^2\hbar^2}\left(\frac{1}{2}\ln\frac{\sqrt{E_C}+\sqrt{\varepsilon_F}}{\sqrt{E_C}-\sqrt{\varepsilon_F}} - \sqrt{\frac{E_C}{\varepsilon_F}}\right), \tag{9}$$

where the logarithm term results from the sum over Bogoliubov modes corresponding to energy above the cut-off E_C performed in the spirit of the local density approximation, see [19] for details. For the simulations we choose $L_z = 40k_F^{-1}$, $L_\perp = 20k_F^{-1}$, $\mu = 0.83\varepsilon_F$ and $k_F|a| = 0.4$ which for $\Gamma = 0$ and the cut-off $E_C = 100\varepsilon_F$ leads to $\Delta_0(T = 0) = 0.036\varepsilon_F$ and $T_c = 0.019T_F$. Using these parameters $l_c \sim 100k_F^{-1}$ is larger than the system size and we are able to explore a regime beyond GL theory. We assume real $\Gamma(\mathbf{r})$ given by a pseudo-random function that changes along z direction only,

$$\Gamma(\mathbf{r}) = \frac{\Gamma_0}{2}\left[\sin\left(\frac{2\pi}{L_z}(9z + 8.8)\right) + \sin\left(\frac{2\pi}{L_z}(13z + 3.6)\right)\right]. \tag{10}$$

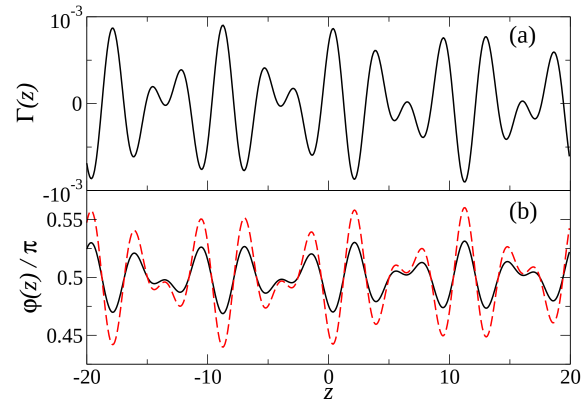

Figure 1: Panel (a) shows $\Gamma(z)$ given in Eq. (10) for $\Gamma_0 = 0.01|\Delta_0(0)|$. Panel (b) presents the corresponding phase $\varphi(z)$ of the pairing function for $T = 0$ (black solid curve) and $T = 0.9T_c$ (red dashed curve).

In Fig. 1 we show the phase of the pairing function $\varphi(z)$ in the case when $\Gamma_0 = 0.01|\Delta_0(0)|$ and $\varphi_\Gamma = 0$ for two different temperatures, i.e. $T = 0$ and $T = 0.9T_c$. One can see that indeed the phase oscillates around $\pi/2$ with a small amplitude (standard deviation of the order of 10^{-2}). The fluctuations of the absolute value of $\Delta(z)$ are negligible (standard deviations divided by average values are of the order 10^{-4}). When T approaches T_c the average $|\Delta|$ decreases and, at some T, becomes much smaller than Γ_0 and we enter another regime where the transfer term in the Hamiltonian (2) starts dominating. For a very large Γ_0 we may expect that a real-valued Δ, which fluctuates out of the phase of $\Gamma(z)$, minimizes the thermodynamic potential. In Fig. 2 we present average values and standard deviations for φ and $|\Delta|$ versus temperature where one can observe an increase of the fluctuations for $T \to T_c$.

In summary, we have shown how to control the relative phase φ between the wavefunction of a molecular condensate and the pairing function of a mixture of fermions in the BCS state. It turns out that a certain class of weak couplings which transfer pairs of fermions into molecules and vice versa, fix this relative phase. The couplings need to vary randomly, or in an oscillatory manner in space; they can be realized by optical means, with a desired phase and amplitude, which in turn allows for efficient control of φ. In this letter we have considered the Fermi system in a weak coupling regime but similar behavior is expected in the strong regime. In particular, translation of our results to the simplified resonant superfluidity theory (cf. [33]) is straightforward. Our results hold also for $0 < k_F a \ll 1$, where the pairing function becomes a condensate wavefunction of tightly bound pairs and we deal with the control of relative phase between two Bose-

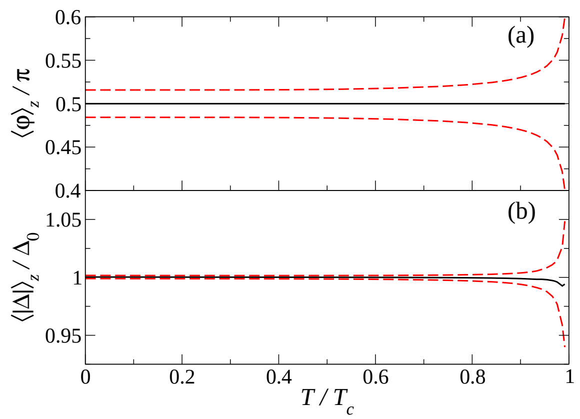

Figure 2: Panel (a) shows average value of the phase of the pairing function $\langle\varphi\rangle_z$ versus temperature obtained for Γ as in Fig. 1. In panel (b) we present the corresponding average value of the modulus of the pairing function $\langle|\Delta|\rangle_z$ divided by $\Delta_0(T)$, i.e. the pairing for the $\Gamma = 0$ case. Solid black curves are related to average values, dashed red curves to average values $\pm$ standard deviation. The figure shows simulations for temperatures up to $T = 0.99T_c$ where $\Delta_0(T) = 0.16\Delta_0(0)$.

Einstein condensates, analyzed in our earlier publication [12]. The problem considered here belongs to a general effect of disorder-induced order phenomena, that rely on continuous symmetry breaking.

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